Probe D–branes in Superconformal Field Theories

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We overview the main configurations of D–brane probes in the $AdS_5 \times X^5$ background of type IIB string theory (X^5 being a Sasaki–Einstein manifold), and examine their most salient features from the point of view of the dual quiver superconformal field theory.

1 Introduction

The open/closed string descriptions of a system of N_c parallel D3–branes in flat space decouple in the large N_c limit, this leading to the duality between $\mathcal{N} = 4$ supersymmetric Yang–Mills theory and type IIB string theory in $AdS_5 \times S^5$ [1]. If, instead, the transverse six dimensional flat space background is replaced by a Calabi–Yau threefold, Y^6, the amount of preserved supercharges reduces to one quarter. [1] Provided Y^6 is a cone on a Sasaki–Einstein manifold X^5, and the stack of D3–branes is placed at the tip of the Calabi–Yau cone $\mathcal{C}(X^5)$, a duality between quiver $\mathcal{N} = 1$ superconformal field theories (SCFT) and type IIB string theory in $AdS_5 \times X^5$ arises [4]. The case in which Y^6 is also toric, and X^5 is topologically $S^2 \times S^3$, is by now very well understood. There are three possibilities [2] (whose main features are displayed in Table 1):

- $X^5 = T^{1,1}$: Its isometry is $SU(2) \times SU(2) \times U(1)$. Its metric has been constructed long ago [5]. The dual gauge theory was worked out soon after the advent of the AdS/CFT correspondence [6].
- $X^5 = Y^{p,q}$: Its isometry is $SU(2) \times U(1) \times U(1)$. Its metric has been discovered more recently [7]. It depends on two positive integers p and q. The dual gauge theory was puzzled out in [8].
- $X^5 = L^{a,b,c}$: Its isometry is $U(1) \times U(1) \times U(1)$. Its metric was found much more recently [9]. It depends on three positive integers a, b and c. The dual SCFT was unraveled in [10].

A crucial ingredient of the AdS/CFT duality is the state/operator correspondence: chiral operators of the CFT are associated with supergravity modes in the dual background. Still, there are features of the gauge theory whose description demands the introduction of (wrapped) D–branes in the gravity side. Most notably, *dibaryon operators* corresponding to each bifundamental chiral field of these quiver gauge theories. They are given by D3–branes wrapping supersymmetric 3-cycles in X^5 [11, 12, 13]. These are point-like objects from the SCFT point of view. This is also the case for the *baryon vertex*, corresponding to a baryon built out of external quarks, that is represented by a D5–brane wrapping X^5 [11].

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1 For the sake of reducing supersymmetry (while also giving up conformal invariance), another avenue involving higher dimensional D–branes wrapping supersymmetric cycles of Y^6 has been explored [2] (see [3] for a recent review with updated references).

2 It is convenient to clarify at this point that they are not independent: $Y^{p,q}$ happens to be a subfamily of $L^{a,b,c}$ (indeed, $Y^{p,q} = L^{p-q,p+q,p}$), and –an orbifold of– $T^{1,1}$ can be obtained as a singular limit of $Y^{p,q}$ (meaningly, $Y^{p,0} = T^{1,1}/\mathbb{Z}_p$).

X^5	Isometry/Global Symmetry	Bifundamental Chiral Fields	N_{gg}
$T^{1,1}$	$SU(2)\times\widetilde{SU}(2)\times U(1)$	$A^{\alpha\,[1]}, \tilde{B}^{\alpha\,[-1]}$	2
$Y^{p,q}$	$SU(2)\times U(1)\times U(1)$	$U_p^{\alpha\,[-p]}, V_q^{\alpha\,[q]}, Y_{p+q}^{[p-q]}, Z_{p-q}^{[p+q]}$	$2p$
$L^{a,b,c}$	$U(1)\times U(1)\times U(1)$	$S_{a+b-c}^{[-c]}, T_c^{[c-a-b]}, W_{c-a}^{[b-c]}, X_{b-c}^{[c-a]}, Y_b^{[a]}, Z_a^{[b]}$	$a+b$

Table 1 Data corresponding to the quiver $\mathcal{N}=1$ theories under discussion. In the third column, there is an upper index α used for doublet fields (with respect to the appropriate $SU(2)$ factor), there is a subindex that indicates the degeneracy (*i.e.*, how many bifundamental chiral fields have the same quantum numbers), and there is an upper label in brackets that displays the $U(1)_B$ charge. The gauge group of each theory is $SU(N_c)\times\cdots\times SU(N_c)$, N_{gg} times.

Extended objects in the gauge theory side also correspond to wrapped D–branes in the string theory side. String-like objects as *confining* or *fat strings* arise from D3–branes wrapping 2-cycles. *Domain walls*, *fractional branes* and *defect CFTs* are given by D5–branes wrapping 2-cycles in X^5. The introduction of *matter hypermultiples* –that is, quarks in the fundamental representation–, requires spacetime filling wrapped D7–branes [14]. If the number of wrapped D–branes is much less than N_c, we can stick to the probe approximation. For instance, this is the case when matter is introduced in the quenched approximation, $N_f \ll N_c$. This is the framework considered in the present talk.

2 Some geometrical facts

Let us consider a solution of IIB supergravity whose metric is of the form $ds^2 = ds^2_{AdS_5} + ds^2_{X^5}$ (we choose, for simplicity, a unit radius $L=1$ for both spaces). The metrics $ds^2_{X^5}$ can be locally written as

$$ds^2_{X^5} \;=\; ds^2_4 + \left[\frac{1}{3}d\psi + \sigma\right]^2 \;, \tag{1}$$

where ds^2_4 is a Kähler–Einstein metric with Kähler form $J_4 = \frac{1}{2}d\sigma$. It is natural to introduce the following vielbein basis in Y^6, $\{dr, e^a, e^5\}$, $a = 1\ldots 4$, such that, for example, $J_4 = e^1\wedge e^2 - e^3\wedge e^4$, and the holomorphic 2-form reads $\Omega_4 = (e^1+ie^2)\wedge(e^3+ie^4)$. A set of local complex coordinates, $\{z_1, z_2, z_3\}$, can be identified, such that the holomorphic 3-form reads

$$\Omega = e^{i\psi}\, r^2\, \Omega_4 \,\wedge \left[\, dr + \, i\, r\, e^5 \,\right] = \frac{dz_1\wedge dz_2 \wedge dz_3}{z_1 z_2} \;. \tag{2}$$

The Killing spinors in these Sasaki–Einstein manifolds read ($\Gamma_* \equiv i\,\Gamma_{x^0x^1x^2x^3}$)

$$\epsilon \;=\; e^{-\frac{i}{2}\tilde{\psi}}\, r^{-\frac{\Gamma_*}{2}}\; \left(1 \,+\, \frac{1}{2}\, x^\alpha\, \Gamma_{rx^\alpha}\;(1\,+\,\Gamma_*\,)\right)\, \eta \;, \tag{3}$$

where $\Gamma_{12}\,\eta = -i\eta$, $\Gamma_{34}\,\eta = i\eta$, and $\tilde{\psi}$ is the angle conjugated to the $U(1)$ R–symmetry.

Consider a Dp–brane probe in $AdS_5\times X^5$. The embedding can be characterized by the set of functions $X^M(\xi^\mu)$, where ξ^μ are the worldvolume coordinates. The supersymmetric embeddings are obtained by imposing the condition $\Gamma_\kappa\,\epsilon = \epsilon$, where ϵ is a Killing spinor of the background [15], and Γ_κ is a matrix that depends on the embedding [16]. Thus, $\Gamma_\kappa\,\epsilon = \epsilon$ is a new projection giving rise to BPS equations that determine the supersymmetric embeddings of the brane probes. It is a local condition that must be satisfied at any point of the probe worldvolume.

3 Dibaryon operators

Dibaryon operators can be built for the different bifundamental fields in the quiver gauge theory. They are pointlike objects that correspond to supersymmetric configurations of D3–branes wrapping a three-cycle, $\mathcal{C}_3 \subset X^5$. The homology of these manifolds allows for several inequivalent cycles. It is important to distinguish between *doublet* and *singlet* dibaryon operators according to the transformation properties of the corresponding constituent chiral field under the global $SU(2)$ symmetry. The conformal dimension Δ of the operator dual to a D3–brane probe wrapping $\mathcal{C}_3$, is proportional to the volume,

$$\Delta(\mathcal{C}_3) \;=\; \frac{\pi}{2}\, N_c\, \frac{\mathrm{Vol}(\mathcal{C}_3)}{\mathrm{Vol}(X^5)}\;. \tag{4}$$

We can then compute the R-charge of the operator since it is related to its dimension, $R = \frac{2}{3}\Delta$. Its baryon number (in units of N_c) can be obtained as the integral of the pullback of a $(2,1)$-form [17],

$$\mathcal{B}(\mathcal{C}_3) = \pm\, i\, k_{X^5}\, \int_{\mathcal{C}_3} P\Big[\Big(\frac{dr}{r} + i\, e^5\Big)\wedge\omega\Big]_{\mathcal{C}_3}\;, \tag{5}$$

where ω is a selfdual (1,1)-form satisfying $d\omega = \omega\wedge J_4 = 0$, and k_{X^5} is a constant that depends on X^5. Armed with these expressions, we can extract all the relevant gauge theory information.

An exhaustive study of different D3–branes embeddings corresponding to all possible dibaryons has been carried out for $T^{1,1}$ [18], $Y^{p,q}$ [19] and $L^{a,b,c}$ [20] superconformal field theories. This was done by demanding κ–symmetry. Besides implying a new projection on the Killing spinor, $\Gamma_\kappa\,\epsilon \;=\; \epsilon$ also entails a set of first order BPS equations whose simplest solutions yield a panoply of embeddings of $\mathcal{C}_3$. Compatibility with the AdS structure of the spinor implies that the D3–brane must be placed at the center of AdS_5. These $1/8$ supersymmetric configurations correspond to dibaryons in the gauge theory side. This assertion can be checked by computing their associated R-charges and baryon numbers.

It is not difficult to show that more general embeddings result from the BPS equations. Indeed, it is possible to show that these are equivalent to Cauchy–Riemann equations for the local complex coordinates z_1 and z_2. Then, the most general solution is given by a holomorphic embedding, $z_2 \;=\; \mathcal{F}(z_1)$. An immediate check consists in realizing that these generalized embeddings are calibrated,

$$P\Big[\,\frac{1}{2}\,J\wedge J\,\Big]_{\mathcal{D}_4} \;=\; \mathrm{Vol}(\mathcal{D}_4)\;, \tag{6}$$

where $\mathrm{Vol}(\mathcal{D}_4) \;=\; r^3\, dr\wedge \mathrm{Vol}(\mathcal{C}_3)$ is the volume form of the divisor. Some of these embeddings can be understood as excitations of the dibaryons in the case of $T^{1,1}$ [18]. However, it is important to stress that this local condition does not always make sense globally. We have seen examples of this feature in $Y^{p,q}$ [19] and $L^{a,b,c}$ [20]. [3]

Excitations of a singlet dibaryon can be represented as graviton fluctuations in the presence of the dibaryon. Instead, certain BPS excitations of the wrapped D3–branes corresponding to doublet dibaryons can be interpreted as a single particle state in AdS_5 [13]. These excitations, roughly speaking, correspond to the insertion of a mesonic operator $\mathcal{O}$. Thus, we have to count all possible inequivalent (in the chiral ring) mesonic operators. They correspond to (short and long) loops in the quiver diagram [21]. The simplest ones are operators with R-charge 2, given by short loops in the quiver. These are the terms appearing in the tree level superpotential. They are all equivalent in the chiral ring. Let us call its representative $\mathcal{O}_1$. It is a spin 1 chiral operator with scaling dimension $\Delta = 3$. Its $U(1)_F$ charge vanishes.

As for the long loops in the quiver, let us focus in the only non-trivial case, $X^5 \;=\; Y^{p,q}$. There are two operators $\mathcal{O}_2$ and $\mathcal{O}_3$ with spin, respectively, $\frac{p+q}{2}$ and $\frac{p-q}{2}$. They have a nonvanishing $U(1)_F$ charge. These are the building blocks of other chiral operators, $\mathcal{O} \;=\; \prod_{i=1}^{3}\mathcal{O}_i^{\,n_i}$. The spectrum of fluctuations

[3] We skip all the details due to space limitations of these Proceedings. We encourage the interested reader who is in quest of subtleties and technicalities to look at the references [18, 19, 20].

of these dibaryons along the transverse S^2 can be worked out [19]. The action for the D3–brane should be expanded around the static configuration, $g = g_{(0)} + \delta g$. At quadratic order, it is possible to identify ground state solutions with BPS operators. Their conformal dimensions can be read off, and the spectrum can be shown to coincide with the mesonic chiral operator quantum numbers.

4 The baryon vertex

For a D5–brane that wraps the whole X^5 space, the flux of the RR $F^{(5)}$–form acts as a source for the electric worldvolume gauge field which, in turn, gives rise to a bundle of fundamental strings emanating from the D–brane. The probe action must include the worldvolume field F in both the Dirac–Born–Infeld (DBI) and Wess–Zumino (WZ) terms:

$$S \,=\, -T_5 \int d^6\xi \,\sqrt{-\det(g+F)} \,+\, T_5 \int d^6\xi \;\; A\wedge F^{(5)} \; . \tag{7}$$

We were unable to find a supersymmetric configuration. From the point of view of κ–symmetry, it turns out that the new projection, $\Gamma_\kappa\,\epsilon = \Gamma_{x^0 r}\,\epsilon^* = \epsilon$, which, as expected, corresponds to fundamental strings in the radial direction, cannot be imposed on the Killing spinors. Besides, it is also possible to show that, from the point of view of the worldvolume theory, there are no solitons saturating a Bogomol'nyi bound. Thus, we conclude from this incompatibility argument that the baryon vertex configuration breaks completely the supersymmetry of the $AdS_5 \times X^5$ background.

5 Fractional brane

Consider a D5–brane probe that wraps a two-dimensional submanifold L_2 of X^5 and is a codimension one object in AdS_5. In the field theory side, this is the kind of brane that represents a domain wall across which the rank of the gauge groups jumps. The upshot of the detailed analysis accomplished in [18, 19, 20] is as follows. We have shown that the cone $\mathcal{L}_3 = \mathcal{C}(L_2)$, is calibrated. Indeed, the holomorphic $(3,0)$ form Ω of $\mathcal{C}(X^5)$ –see eq.(2)–, can be naturally used to calibrate such submanifolds: $\mathcal{L}_3$ is called a special Lagrangian submanifold of $\mathcal{C}(X^5)$ if the pullback of Ω to $\mathcal{L}_3$ is, up to a constant phase λ, equal to its volume,

$$P\big[\,\Omega\,\big]_{\mathcal{L}_3} \,=\, e^{i\lambda}\,\mathrm{Vol}\,(\mathcal{L}_3) \; . \tag{8}$$

The fractional brane can be also understood as a BPS worldvolume soliton. This arises from the Hamiltonian density resulting from the DBI action, since it can be written as a sum of squares in such a way that it becomes minimum when a set of BPS differential equations are satisfied. Not surprisingly, they agree with those obtained from the κ–symmetry approach.

6 Flavor D7–branes

The D7-branes which fill the four Minkowski spacetime dimensions and extend along some holographic non-compact direction can be potentially used as flavor branes, *i.e.* as branes whose fluctuations can be identified with the dynamical mesons of the gauge theory [14]. The ansatz we adopt for the worldvolume coordinates is $\xi^\mu = (x^\alpha, \theta^\beta)$, and we consider embeddings of the form $r = r(\theta^\beta)$ and $\psi = \psi(\theta^\beta)$. In order to implement $\Gamma_\kappa\,\epsilon = \epsilon$, we shall require that the spinor ϵ is an eigenvector of the matrix Γ_* defined above. These configurations preserve the four ordinary supersymmetries of the background. By means of the κ–symmetry technique, it is possible to show that a generic configuration can be nicely written as a holomorphic embedding [18, 19, 20]

$$z_1^{m_1}\, z_2^{m_2}\, z_3^{m_3} \,=\, \text{constant} \; , \tag{9}$$

where the m_i's are constants and $m_3 \neq 0$. These configurations seem to be the relevant ones to introduce matter in the fundamental representation [22].

The identification of supersymmetric 4-cycles that a D7–brane can wrap also matters in cosmological models where inflation is produced by the motion of a D3–brane in a warped throat. The potential ruling this motion is actually sensitive to the specific embedding of the wrapped D7–brane [23].

7 Further configurations

Another cases of interest include a non-supersymmetric (still stable) probe D3–brane extended along one gauge theory direction and wrapping a 2-cycle (a *fat string* from the gauge theory point of view), a D5–brane that extends infinitely in the holographic direction (a *defect CFT* that preserves four supersymmetries), and a D7–brane wrapping the whole X^5 space, and being codimension two in AdS_5 (a supersymmetric *string*). In the D5–brane configuration, we have also turned on a worldvolume flux and found that it leads to a bending of the profile of the wall.

Details can be found in the original references [18, 19, 20].

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References

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
[2] J. M. Maldacena and C. Nunez, Phys. Rev. Lett. **86**, 588 (2001).
[3] J. D. Edelstein and R. Portugues, Fortsch. Phys. **54**, 525 (2006).
[4] S. S. Gubser, Phys. Rev. D **59**, 025006 (1999).
[5] P. Candelas and X. C. de la Ossa, Nucl. Phys. B **342**, 246 (1990).
[6] I. R. Klebanov and E. Witten, Nucl. Phys. B **536**, 199 (1998).
[7] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, Class. Quant. Grav. **21**, 4335 (2004); Adv. Theor. Math. Phys. **8**, 711 (2004).
[8] D. Martelli and J. Sparks, Commun. Math. Phys. **262**, 51 (2006). S. Benvenuti, S. Franco, A. Hanany, D. Martelli and J. Sparks, JHEP **0506**, 064 (2005).
[9] M. Cvetic, H. Lu, D. N. Page and C. N. Pope, Phys. Rev. Lett. **95**, 071101 (2005). D. Martelli and J. Sparks, Phys. Lett. B **621**, 208 (2005).
[10] S. Benvenuti and M. Kruczenski, JHEP **0604**, 033 (2006). S. Franco, A. Hanany, D. Martelli, J. Sparks, D. Vegh and B. Wecht, JHEP **0601**, 128 (2006). A. Butti, D. Forcella and A. Zaffaroni, JHEP **0509**, 018 (2005).
[11] E. Witten, JHEP **9807**, 006 (1998).
[12] S. S. Gubser and I. R. Klebanov, Phys. Rev. D **58**, 125025 (1998). S. Gukov, M. Rangamani and E. Witten, JHEP **9812**, 025 (1998). C. E. Beasley, JHEP **0211**, 015 (2002).
[13] D. Berenstein, C. P. Herzog and I. R. Klebanov, JHEP **0206**, 047 (2002).
[14] A. Karch and E. Katz, JHEP **0206**, 043 (2002).
[15] K. Becker, M. Becker and A. Strominger, Nucl. Phys. B **456**, 130 (1995). E. Bergshoeff, R. Kallosh, T. Ortin and G. Papadopoulos, Nucl. Phys. B **502**, 149 (1997). E. Bergshoeff and P. K. Townsend, JHEP **9905**, 021 (1999).
[16] M. Cederwall, A. von Gussich, B. E. W. Nilsson, P. Sundell and A. Westerberg, Nucl. Phys. B **490**, 179 (1997). E. Bergshoeff and P. K. Townsend, Nucl. Phys. B **490**, 145 (1997). M. Aganagic, C. Popescu and J. H. Schwarz, Phys. Lett. B **393**, 311 (1997); Nucl. Phys. B **495**, 99 (1997).
[17] C. P. Herzog, Q. J. Ejaz and I. R. Klebanov, JHEP **0502**, 009 (2005).
[18] D. Arean, D. Crooks and A. V. Ramallo, JHEP **0411**, 035 (2004).
[19] F. Canoura, J. D. Edelstein, L. A. P. Zayas, A. V. Ramallo and D. Vaman, JHEP **0603**, 101 (2006).
[20] F. Canoura, J. D. Edelstein and A. V. Ramallo, JHEP **0609**, 038 (2006).
[21] S. Benvenuti and A. Hanany, JHEP **0508**, 024 (2005). S. Benvenuti and M. Kruczenski, JHEP **0610**, 051 (2006).
[22] T. Sakai and J. Sonnenschein, JHEP **0309**, 047 (2003). P. Ouyang, Nucl. Phys. B **699**, 207 (2004). C. Nuñez, A. Paredes and A. V. Ramallo, JHEP **0312**, 024 (2003). T. S. Levi and P. Ouyang, ArXiv:hep-th/0506021.
[23] A. Dymarsky, S. Gubser, Z. Guralnik and J. M. Maldacena, JHEP **0609**, 057 (2006).